Scaling Laws for Age of Information in Wireless Networks

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***Abstract*— We study age of information in a multiple source-multiple destination setting with a focus on its scaling in large wireless networks. There are n nodes uniformly and independently distributed on a fixed area that are randomly paired with each other to form n source-destination (S-D) pairs. Each source node wants to keep its destination node as up-to-date as possible. To accommodate successful communication between all n S-D pairs, we first propose a three-phase transmission scheme which utilizes local cooperation between the nodes along with what we call *mega update packets* to serve multiple S-D pairs at once. We show that under the proposed scheme average age of an S-D pair scales as $O(n^{\frac{1}{4}}\log n)$ as the number of users, n, in the network grows. Next, we observe that communications that take place in Phases I and III of the proposed scheme are scaled-down versions of network-level communications. With this along with scale-invariance of the system, we introduce hierarchy to improve this scaling result and show that when *hierarchical cooperation* between users is utilized, an average age scaling of $O(n^{\alpha(h)}\log n)$ per-user is achievable, where h denotes the number of hierarchy levels and $\alpha(h) = \frac{1}{3\cdot 2^h+1}$. We note that $\alpha(h)$ tends to 0 as h increases, and asymptotically, the average age scaling of the proposed hierarchical scheme is $O(\log n)$. To the best of our knowledge, this is the best average age scaling result in a status update system with multiple S-D pairs.**

***Index Terms*— Age of information, scaling laws, large networks, hierarchical cooperation.**

I. Introduction

FROM smart homes to stock market and unmanned aerial vehicle (UAV) systems many recent applications involve real-time monitoring of a phenomenon of interest. In these applications, resulting measurements obtained by the sources are sent to the interested recipient nodes in the form of status update packets. Common to all these applications is the need of timely delivery of these packets since more recent measurements better capture the source process. In other words, in these systems information loses its value as it becomes stale. In order to measure the freshness of the received information, age of information (AoI) metric has been proposed. Age tracks the time elapsed since the most recent update packet at the destination node was generated at the source node. In other words, at time t, age $\Delta(t)$ of a packet which has a timestamp $u(t)$ is $\Delta(t) = t - u(t)$. Age of information has been extensively studied in the literature in the context of queueing networks [3]–[13], social networks [14], remote estimation [15]–[18], scheduling in networks [19]–[22], energy harvesting systems [23]–[34], source coding [35]–[40], vehicular, IoT and UAV systems [41]–[44], reinforcement learning [45]–[47], and so on. Common to all these works is the fact that they study the analysis and optimization of age of information in systems with small number of source-destination pairs. In this work, unlike prior works, our aim is to analyze the age of information in a large network setting with many source-destination pairs with a focus on its scalability as a function of the network size.

With increasing connectivity in communication networks and rapidly growing number of information sources (both people and sensors), the issue of scalability of age of information has emerged. In early 00's, following the pioneering work of Gupta and Kumar [48], a similar issue had come up for scaling laws of *throughput* in large networks. Reference [48] uses a multi-hop scheme that achieves a total throughput of $O(\sqrt{n})$ for the network, and hence, $O(\frac{1}{\sqrt{n}})$ throughput per-user. References [49]–[52] studied throughput scaling in dense and extended networks considering static and mobile nodes. This line of research has culminated in the seminal papers of Ozgur *et al.* [53], [54] which achieved $O(1)$ throughput per-user by utilizing hierarchical cooperation between nodes. In this paper, we study scaling of *age of information* in large wireless networks.

What makes age analysis in large networks challenging is the fact that good age performance corresponds to neither high throughput nor low delay. As argued in references [55] and [56], for the optimized age performance we need regular packet delivery with low delay. The way to achieve the maximum throughput is to send as many updates as possible from the source. However, this may cause congestion in the system resulting in stale packet deliveries at the destination. Likewise,

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packet delay in the network can be reduced by decreasing the update frequency which in turn yields outdated information at the destination since the update delivery rate is low. In this paper, we balance these two opposing objectives, and develop an achievable scheme that strikes a balance between the two in large networks.

References that are most closely related to our work study the scaling of age of information in the broadcast setting [14], [57]–[60]. These works study a single source node which sends status updates to multiple receiver nodes. Reference [14] studies a mobile social network with a single service provider and n communicating users, and shows that under Poisson *contact processes* among users and uniform rate allocation from the service provider, the average age of the content at the users is $O(\log n)$. Without the contact process between users, however, age grows linearly in n since the service provider serves only one user at a time. In [57]–[60], single and multihop multicast networks are considered and $O(1)$ average age is obtained at the end nodes by using special transmission schemes such as the earliest k transmission scheme in which the source node waits for delivery to the earliest k out of the total n receiver nodes. Reference [61], on the other hand, studies age scaling in the multiaccess setting with a massive number of source nodes.

In this work, we focus on a multiple source-multiple destination setting and study a fixed area network of n randomly located source-destination (S-D) pairs that want to send time-sensitive update packets to each other. Each node is both a source and a destination. We aim to find a transmission scheme which allows all n S-D pairs to successfully communicate and achieves the smallest average age scaling per-user.

As studied in [61], a straightforward way to achieve successful communication between all S-D pairs is to use a round-robin policy such that at each turn only one source transmits to its destination and stays silent while all other sources transmit successively during their respective turns. This direct method achieves an age scaling of $O(n)$ meaning that age increases linearly in n since under this policy average inter-update times at a destination node increases linearly as n grows making the updates less frequent and causing age to increase.

As in the setting of [48], a multihop scheme that involves successive transmissions between the source and destination nodes can be utilized. In that work, the network is divided into cells and transmission hops take place in between these cells such that $O(\sqrt{n})$ messages are carried by each cell. Each of these cells can be considered a queue with multiple sources. As studied in [62], the age of a single update packet that is served by a queue with $O(\sqrt{n})$ different packet streams is also $O(\sqrt{n})$ under LCFS with preemption policy. Therefore, in the multihop scheme, after one hop, age of an update becomes $O(\sqrt{n})$ since the queue is shared by $O(\sqrt{n})$ other packets. Considering the fact that the number of hops needed is $O(\sqrt{n})$, using a multihop scheme, the average age scales as $O(n)$ as in [61].

In this paper, considering all these previous results, we first propose a three-phase transmission scheme to serve all n S-D pairs such that the time average age of each node is small. Our scheme utilizes local cooperation between the users as in [53]. We divide the network into cells of M users each. In the first phase, nodes from the same cell communicate to create a *mega update packet*, which contains the updates of all nodes from that cell. In the second phase, inter-cell communication takes place and each cell sends its mega packet to the corresponding destination cells. The main idea behind the mega update packets is to serve many nodes at once to decrease inter update time. In the third and final phase, individual packets are extracted from the received mega update packets and relayed to the intended recipient nodes in the cells. During all these phases, we make use of the spatial separation of the nodes to allow multiple simultaneous transmissions provided that there is no destructive interference caused by others.[1] Using this scheme, we achieve an average age scaling of $O(n^{\frac{1}{4}}\log n)$ per-user.

Next, we observe that the first and third phases of the proposed transmission scheme essentially require successful communication between pairs but among M nodes rather than n. With this observation and the fact that the system is scale-invariant, we introduce hierarchy in Phases I and III to improve the age scaling result. In other words, we can further divide cells into smaller subcells and apply the proposed three-phase transmission scheme again in Phases I and III. Although hierarchical cooperation was shown to result in poor delay performance in [54], by utilizing mega update packets better age scaling can be achieved here. In fact, using this scheme, we show that an average age scaling of $O\left(n^{\alpha(h)}\log n\right)$ per-user is achievable where $\alpha(h) = \frac{1}{3\cdot 2^h+1}$ and h denotes the number of hierarchy levels. We note that when this hierarchical cooperation is not utilized, i.e., $h = 0$, we retrieve the performance of the initial scheme which achieves an age scaling of $O(n^{\frac{1}{4}}\log n)$ per-user. In the asymptotic case when $h \to \infty$, the proposed scheme with hierarchical cooperation achieves an average age scaling of $O(\log n)$. To the best of our knowledge, this is the best average age scaling result in a status update system with multiple S-D pairs.

II. System Model and Age Metric

We consider n nodes that are uniformly and independently distributed on a square of fixed area S. Every node is both a source and a destination. These sources and destinations are paired randomly irrespective of their locations to form n S-D pairs. Sources create time-sensitive status update packets and transmit them to their respective destinations using the common wireless channel. Each source wants to keep its destination as up-to-date as possible. Thus, destination nodes need to be updated regularly with low transmission delays. We use the age of information metric to measure the freshness of the status update packets. Age is measured for each destination node and for node i at time t age is the random process $\Delta_i(t) = t - u_i(t)$ where $u_i(t)$ is the timestamp of the most

[1] We show in Section VI that the effect of the interference under the protocol model [48] is a scaling constant; see Section VI for details.

recent update at that node. The metric we use, time averaged age, is

$$\Delta_i = \lim_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau \Delta_i(t)dt, \tag{1}$$

for node i. We use a graphical average age analysis to derive the average age for a single S-D pair assuming ergodicity similar to [3] and [9].

Inspired by [53], we first propose a scheme based on clustering nodes and making use of what we call *mega update packets* to increase the spatial reuse of the common wireless channel. This entails dividing n users into $\frac{n}{M}$ cells with M users in each cell with high probability.[2] The users communicate locally within cells to form the mega update packets. We model the delay in these intra-cell communications as i.i.d. exponential with parameter λ. Then, mega packets are transmitted between the cells. We model the delay in these inter-cell communications as i.i.d. exponential with parameter $\tilde{\lambda}$. Finally, the individual updates are extracted from mega updates and distributed to the intended destinations within cells again via intra-cell communications. While intra-cell communications occur simultaneously in parallel across the cells (see Section VI for details), inter-cell updates occur sequentially one-at-a-time.

Second, we observe that the bottleneck in average age scaling in this three-phase scheme is M since during intra-cell transmissions M transmissions are needed, one for each node in a cell. Noting that each cell is a scaled-down version of the whole system, we then propose introducing hierarchy by forming subcells from the cells and applying the three-phase transmission scheme on a cell level to overcome this bottleneck. Thus, when $h = 1$ hierarchy level is utilized, this hierarchical scheme includes inter-cell, inter-subcell (within cells) and intra-subcell transmissions. We first analyze the case with $h = 1$ level of hierarchy and then generalize the result to h hierarchy levels. We again model the delay in communications as i.i.d. exponential random variables with varying parameters depending on the type, e.g., intra-subcell, inter-subcell within cells or inter-cell, of the communication (see Section V for details).

Due to i.i.d. nature of delivery times in all types of communications with or without hierarchy, all destination nodes experience statistically identical age processes and will have the same average age. Thus, we will drop user index i in the average age expression and use Δ instead of Δ_i in the following analysis.

Finally, we denote the kth order statistic of random variables $X_1, \ldots, X_n$ as $X_{k:n}$. Here, $X_{k:n}$ is the kth smallest random variable, e.g., $X_{1:n} = \min\{X_i\}$ and $X_{n:n} = \max\{X_i\}$. For i.i.d. exponential random variables X_i with parameter λ, we have [63]

$$\mathbb{E}[X_{k:n}] = \frac{1}{\lambda}(H_n - H_{n-k}), \tag{2}$$

[2]As shown in [53, Lemma 4.1], the probability of having $((1-\delta)M, (1+\delta)M)$ nodes in a cell is larger than $1 - \frac{n}{SM}e^{-\Lambda(\delta)M}$ where $\Lambda(\delta)$ is independent of n and satisfies $\Lambda(\delta) > 0$ when $\delta > 0$. Note that when $M = n^b$ with $0 < b \le 1$ this probability tends to 1 as n grows.

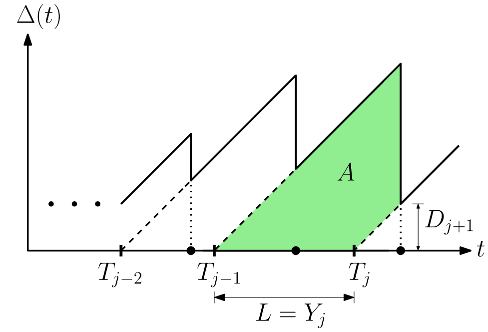

Fig. 1. Sample age $\Delta(t)$ evolution for a single S-D pair. Update deliveries are shown with symbol $\bullet$. Session j starts at time T_{j-1} and lasts until $T_j = Y_j + T_{j-1}$.

$$\mathrm{Var}[X_{k:n}] = \frac{1}{\lambda^2}(G_n - G_{n-k}), \tag{3}$$

where $H_n = \sum_{j=1}^n \frac{1}{j}$ and $G_n = \sum_{j=1}^n \frac{1}{j^2}$. Using these,

$$\mathbb{E}[X_{k:n}^2] = \frac{1}{\lambda^2}\left((H_n - H_{n-k})^2 + G_n - G_{n-k}\right). \tag{4}$$

Note that, for large n, we have $H_n \approx \log n + \gamma$ and $G_n \to \frac{\pi^2}{6}$ where $\gamma \approx 0.577$ is the Euler-Mascheroni constant. Since the constant γ does not affect the scaling results presented in this paper, we take $H_n \approx \log n$ for large n in the rest of the paper for ease of exposition. Throughout the paper, the equality of two random variables stands for the equality of these random variables in distribution. Further, the inequalities involving random variables correspond to the usual stochastic order of random variables. That is, given random variables X and Y, X is stochastically smaller than Y if $P(X > x) \le P(Y > x), \forall x \in \mathbb{R}$ [64].

III. Age Analysis of a Single S-D Pair

The network operates in sessions such that during each session all n sources successfully send their update packets to their corresponding destinations. Each session lasts for Y units of time. Here, we derive the average age of a single S-D pair (s, d) since each pair experiences statistically identical age as explained in Section II.

Session j starts at time T_{j-1} and all sources including s generate their respective jth update packets. This session lasts until time $T_j = T_{j-1} + Y_j$, at which point, all n packets are received by their designated recipient nodes including node d. In other words, a session ends when the last S-D pair finishes the packet transmission at which point, all the other $n-1$ destination nodes have already received their packets. Thus, in the proposed scheme every destination node but one receives its packet before the session ends. Fig. 1 shows the evolution of the age at a destination node over time. It is in the usual sawtooth shape with the age increasing linearly over time and dropping to a smaller value as the updates are received at the destination. The process repeats itself at time T_j when all sources including s generate the next update packet, namely update $j+1$.

Using Fig. 1, the average age for an S-D pair is given by

$$\Delta = \frac{\mathbb{E}[A]}{\mathbb{E}[L]}, \tag{5}$$

where A denotes the shaded area and L is its length. From the figure, we find $A_j = \frac{1}{2}Y_j^2 + Y_j D_{j+1}$ such that

$$\mathbb{E}[A] = \frac{\mathbb{E}[Y^2]}{2} + \mathbb{E}[D]\,\mathbb{E}[Y], \tag{6}$$

$$\mathbb{E}[L] = \mathbb{E}[Y], \tag{7}$$

since the system is ergodic and Y_j and D_{j+1} are independent. Here, D denotes the time interval between the generation of an update and its arrival at the destination node. Using these in (5), the average age for an S-D pair is given by

$$\Delta = \mathbb{E}[D] + \frac{\mathbb{E}[Y^2]}{2\,\mathbb{E}[Y]}. \tag{8}$$

Note that in some systems D may be directly equal to the link delay. However, as in our model here, D may capture some additional delays that may occur during the delivery time of an update. This will be further clarified in the next section.

IV. Three-Phase Transmission Scheme

The proposed scheme involves clustering nodes and making use of *mega update packets* to serve many S-D pairs at once to reduce the session time. In this section, we describe the proposed three-phase transmission scheme and define mega update packets. As in [53], we divide the square network area into $\frac{n}{M}$ cells of equal area such that each cell includes M nodes with high probability which tends to 1 as n increases.[3] The transmission delays between the nodes belonging to the same cell are denoted by X_i whereas the transmission delays between the nodes from different cells are denoted by $\tilde{X}_i$. Note that X_i and $\tilde{X}_i$ are independent; X_i are i.i.d. exponential with parameter λ and $\tilde{X}_i$ are i.i.d. exponential with parameter $\tilde{\lambda}$.[4]

Phase I. Creating Mega Update Packets. In a cell, each one of the M nodes gets a turn to distribute its current update packet to remaining $M-1$ nodes through $M-1$ links with independent random delays. This operation resembles the wait-for-all scheme studied in [57] since each node keeps transmitting until all $M-1$ nodes receive its packet. Thus, the time needed for each node to distribute its update packet to other nodes in the cell is $U = X_{M-1:M-1}$. Considering M successive transmissions for each node in the cell, this phase is completed in $V = \sum_{i=1}^{M} U_i$ units of time. By the end of this phase in a cell, each one of the M nodes has M different update packets one from each other node in that cell. Each node combines all these M packets to create what we call a *mega update packet* (see Fig. 2). In order to reduce the session time, cells work in parallel during Phase I (see Section VI for a detailed description of this operation). This phase ends when the slowest cell among simultaneously operating cells finishes creating its mega update packet. Phase I

[3]We note that it is sufficient to have $O(M)$ nodes in each cell for the proposed scheme to work. However, from hereafter, we assume that each cell has exactly M nodes for ease of exposition.

[4]We note that we have $\tilde{\lambda} \leq \lambda$ to reflect the increased distance and packet size, due to the utilization of mega update packets in the inter-cell transmissions of Phase II. In Section VIII, we take $\tilde{\lambda}$ as a function of M to further account for the mega update packet size in the inter-cell transmission delays of Phase II.

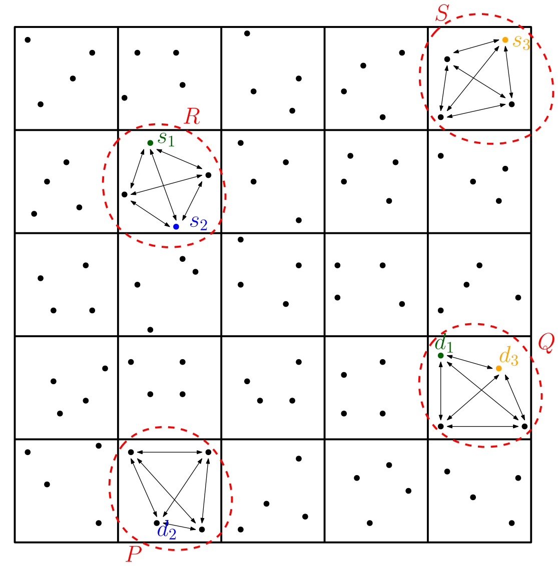

Fig. 2. Cell formation for $M = 4$ and $n = 100$. Simultaneous intra-cell transmissions are depicted for three S-D pairs from cells P, Q, R and S.

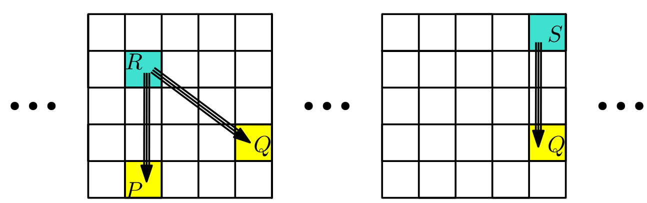

Fig. 3. In Phase II cells take turns to perform inter-cell transmissions. These inter-cell transmissions are shown for the same three S-D pairs depicted in Fig. 2.

takes $Y_I = V_{\frac{n}{M}:\frac{n}{M}}$ units of time, where Y_I denotes the duration of Phase I.

Phase II. MIMO-Like Transmissions In this phase, each cell successively performs MIMO-like transmissions using the mega update packets created in Phase I. In each cell, all M source nodes send the mega update packet through the channel simultaneously to the respective destination cells in which the destination nodes are located. Since every node sends the same mega packet which includes all M packets to be transmitted from that cell, this does not create interference. Thus, this is equivalent to sending update packets of all M sources with M copies each all at once (see Fig. 3). Hence, this significantly reduces the time needed to transmit updates of all M sources from that cell to their respective destinations. Note that this stage does not require the destination nodes to be in the same cell. In fact, considering that we have M nodes in a cell, each cell can at most have M different destination cells. Since we send M copies of each update to a destination cell in which there are M receiver nodes, only the earliest successful transmission is important. In other words, among the M^2 active links only the one with the smallest delay is critical in delivering the mega update packet to a destination cell. Thus, it takes $\tilde{U} = \tilde{X}_{1:M^2}$ units of time for a source node s from cell j to send its update to the destination cell where the destination node d lies in.[5] This MIMO-like transmissions of cell j continues until all M destination cells receive the mega packet. Hence, for each cell, this phase lasts for $\tilde{V} = \tilde{U}_{M:M}$. We repeat this for each cell, making the session time of this phase $Y_{II} = \sum_{i=1}^{\frac{n}{M}} \tilde{V}_i$.

Phase III. In-Cell Relaying to the Destination Nodes By the end of Phase II, each cell receives a mega packet for each one of its nodes. These packets may be received directly by their intended destination nodes. However, considering the worst case where they are received by any other node, we need to relay them to their actual designated recipient nodes. Thus, in this phase, M actual messages are extracted from the corresponding M mega update packets received during Phase II and sent to their recipients one at a time. Since this phase has intra-cell transmissions, it is performed in parallel across cells. For a single node this takes X units of time, consequently we need $\hat{V} = \sum_{i=1}^{M} X_i$ to finish this process in a cell. As in Phase I, we need to wait for the slowest cell to finish this operation. Then, this phase lasts for $Y_{III} = \hat{V}_{\frac{n}{M}:\frac{n}{M}}$ units of time.

The total session time of the proposed scheme is,

$$Y = Y_I + Y_{II} + Y_{III} = V_{\frac{n}{M}:\frac{n}{M}} + \sum_{i=1}^{\frac{n}{M}} \tilde{V}_i + \hat{V}_{\frac{n}{M}:\frac{n}{M}}, \quad (9)$$

where V, $\tilde{V}$, and $\hat{V}$ are defined above. In our proposed scheme, assuming no S-D pair is in the same cell, arrivals to destination nodes occur in Phase III. Note that when an S-D pair is in the same cell, corresponding D is smaller which consequently leads to a smaller age, where as noted earlier, D denotes the time between generation of an update at certain source node till its arrival at the corresponding destination node. Therefore, by assuming no S-D pair is in the same cell, we essentially consider the worst case. Thus, any successful packet delivery will happen no earlier than the duration of the first two phases $Y_I + Y_{II}$. In addition, Phase III involves M successive in-cell transmissions for each node of a particular cell. Hence, depending on the cell that the source node lies in, as well as the realization of the transmission delay X, the corresponding destination node may receive the packet some time after Phase III starts. Let random variable Z capture when after Phase III starts that particular S-D pair is served. Then, we have,

$$D = Y_I + Y_{II} + Z. \quad (10)$$

For example, if a packet is the $(j+1)$th to be transmitted in Phase III, then delivery will be at $Y_I + Y_{II} + \sum_{i=1}^{j} X_i + X$. Then, the random variable Z is of the form $Z = \sum_{i=1}^{j} X_i + X$.

Substituting (9)-(10) in (8) we obtain,

$$\Delta = \mathbb{E}[Y_I] + \mathbb{E}[Y_{II}] + \mathbb{E}[Z] + \frac{\mathbb{E}[Y^2]}{2\,\mathbb{E}[Y]}, \quad (11)$$

which is the average age of an S-D pair under the proposed transmission scheme.

Before we perform the explicit age calculation using (11), we make some observations to simplify our analysis. First, we note that, when the transmission delays $\tilde{X}$ are i.i.d. exponential with rate $\tilde{\lambda}$, then $\tilde{U} = \tilde{X}_{1:M^2}$ is also exponential with rate $M^2\tilde{\lambda}$ [65]. Second, we have the following upper bound for the duration of Phase I.

Lemma 1: Y_I satisfies the following inequality,

$$Y_I \leq \bar{V}, \quad (12)$$

where $\bar{V} = \sum_{i=1}^{M} \bar{U}_i$ and $\bar{U} = X_{n:n}$.

Proof: Recall that $Y_I = V_{\frac{n}{M}:\frac{n}{M}}$, where $V = \sum_{i=1}^{M} U_i$ and $U = X_{M-1:M-1}$. To show the inequality we make the following observation: In Phase I, $\frac{n}{M}$ cells operate simultaneously. First nodes of each of these cells start transmitting their packets to all other $M-1$ nodes of their cell at the same time. Here, the term first nodes denotes the set of arbitrarily selected nodes, one from each cell, that distribute their packet in their respective cells in the first place. Since intra-cell transmission delays are all i.i.d. across cells and packets, what we essentially have in this case is simultaneous transmission to $\frac{n}{M}(M-1) \approx n$ nodes, and therefore all first nodes will be done in $X_{n:n}$ units of time.

We repeat this for the second nodes of each cell, i.e., nodes that distribute their packet within their respective cells in the second place, and so on to get $\bar{V} = \sum_{i=1}^{M} (X_{n:n})_i = \sum_{i=1}^{M} \bar{U}_i$. In this way of operation, a cell waits for all other cells to finish distributing the update packet of the first node and then continues with the second node and so on. In a way, for each of its nodes it waits for the slowest cell to finish. However, in our constructed scheme during Phase I, inside a cell, nodes distribute their packets to other nodes of that cell without considering other cells and phase ends when all cells finish this process for all their M nodes. Thus, $\bar{V}$ is an upper bound on Y_I. ■

Although our proposed Phase I lasts shorter than the scheme described in Lemma 1, for tractability and ease of calculation we worsen our scheme in terms of session time, and take the upper bound in Lemma 1 as our Phase I duration such that from now on $Y_I = \bar{V}$. Third, we have the following upper bound for the duration of Phase III.

Lemma 2: Y_{III} satisfies the following inequality,

$$Y_{III} \leq \bar{\bar{V}}, \quad (13)$$

where $\bar{\bar{V}} = \sum_{i=1}^{M} \bar{\bar{U}}_i$ and $\bar{\bar{U}} = X_{\frac{n}{M}:\frac{n}{M}}$.

We omit the proof of Lemma 2 since it follows similar to the proof of Lemma 1. Due to the same tractability issues, we worsen Phase III as well in terms of duration and take $Y_{III} = \bar{\bar{V}}$ from now on.

As a result of Lemmas 1 and 2, (9) becomes

$$Y = \bar{V} + \sum_{i=1}^{\frac{n}{M}} \tilde{V}_i + \bar{\bar{V}}. \quad (14)$$

Now, we are ready to derive an age expression using Lemmas 1 and 2 in (11). This is stated in the following theorem.

[5] We note that in the MIMO-like transmissions of Phase II, since we consider the earliest transmission among M^2 active links, the speed up factor is M^2 unlike the regular MIMO transmission scheme in which the speed up factor is M. We thank one of the anonymous reviewers for raising this point. In Section VIII, we discuss the performance of the proposed scheme when the speed up factor is only M.

Theorem 1: Under the constructed transmission scheme, the average age of an S-D pair is given by,

$$\begin{aligned}\Delta = &\frac{M}{\lambda}H_n + \frac{n}{M^3\tilde{\lambda}}H_M + \frac{M-1}{2\lambda}H_{\frac{n}{M}} + \frac{1}{\lambda}\\&+\frac{\frac{M^2}{\lambda^2}H_n^2 + \frac{M}{\lambda^2}G_n}{2\left(\frac{M}{\lambda}H_n + \frac{n}{M^3\tilde{\lambda}}H_M + \frac{M}{\lambda}H_{\frac{n}{M}}\right)}\\&+\frac{\frac{n^2}{M^6\tilde{\lambda}^2}H_M^2 + \frac{n}{M^5\tilde{\lambda}^2}G_M}{2\left(\frac{M}{\lambda}H_n + \frac{n}{M^3\tilde{\lambda}}H_M + \frac{M}{\lambda}H_{\frac{n}{M}}\right)}\\&+\frac{\frac{M^2}{\lambda^2}H_{\frac{n}{M}}^2 + \frac{M}{\lambda^2}G_{\frac{n}{M}}}{2\left(\frac{M}{\lambda}H_n + \frac{n}{M^3\tilde{\lambda}}H_M + \frac{M}{\lambda}H_{\frac{n}{M}}\right)}\\&+\frac{\frac{n}{M^2\lambda\tilde{\lambda}}H_nH_M + \frac{M^2}{\lambda^2}H_nH_{\frac{n}{M}} + \frac{n}{M^2\lambda\tilde{\lambda}}H_MH_{\frac{n}{M}}}{\frac{M}{\lambda}H_n + \frac{n}{M^3\tilde{\lambda}}H_M + \frac{M}{\lambda}H_{\frac{n}{M}}}. \quad (15)\end{aligned}$$

Proof: The proof follows upon substituting (14) back in (11) and taking expectations of order statistics of exponential random variables as in Section II. Doing these, we obtain

$$\mathbb{E}[Y_I] = \frac{M}{\lambda}H_n, \quad \mathbb{E}[Y_I^2] = \frac{M^2}{\lambda^2}H_n^2 + \frac{M}{\lambda^2}G_n, \quad (16)$$

$$\mathbb{E}[Y_{II}] = \frac{n}{M^3\tilde{\lambda}}H_M, \quad \mathbb{E}[Y_{II}^2] = \frac{n^2}{M^6\tilde{\lambda}^2}H_M^2 + \frac{n}{M^5\tilde{\lambda}^2}G_M, \quad (17)$$

$$\mathbb{E}[Y_{III}] = \frac{M}{\lambda}H_{\frac{n}{M}}, \quad \mathbb{E}[Y_{III}^2] = \frac{M^2}{\lambda^2}H_{\frac{n}{M}}^2 + \frac{M}{\lambda^2}G_{\frac{n}{M}}. \quad (18)$$

Lastly, we need to calculate $\mathbb{E}[Z]$ where the random variable Z is the additional amount of time after Phase II ends until the destination node receives the update. Let us take an S-D pair (s, d) where source node s is from cell $j+1$. In Phase III, d has to wait for all other j mega packets from the first j cells to be distributed among the nodes. When its turn comes, d just needs X amount of time to get its packet. Then, d has $Z = \sum_{i=1}^{j}\bar{\bar{U}}_i + X$. Here, we have $\bar{\bar{U}}$ inside the summation as opposed to X as in the discussion preceding (11) because of Lemma 2. Taking expectation on $\bar{\bar{U}}$, j and X by noting their mutual independence we get

$$\mathbb{E}[Z] = \left(\frac{1}{M}\sum_{j=0}^{M-1} j\right)\mathbb{E}[\bar{\bar{U}}] + \mathbb{E}[X] = \frac{M-1}{2\lambda}H_{\frac{n}{M}} + \frac{1}{\lambda}. \quad (19)$$

Using (16)-(19) in (11) yields the expression. ■

Having derived the expression for the average age Δ of an S-D pair, we are now ready to work with large n.

Theorem 2: For large n and with $M = n^b$, where $0 < b \leq 1$, the average age Δ in Theorem 1 approximately becomes,

$$\begin{aligned}\Delta \approx &\frac{n^b}{\lambda}\log n + \frac{n}{n^{3b}\tilde{\lambda}}b\log n + \frac{n^b-1}{2\lambda}(1-b)\log n + \frac{1}{\lambda}\\&+\frac{\left(1+(1-b)^2\right)\frac{n^{2b}}{\lambda^2}(\log n)^2}{2\left((2-b)\frac{n^b}{\lambda}\log n + \frac{n}{n^{3b}\tilde{\lambda}}b\log n\right)}\\&+\frac{\frac{n^2}{n^{6b}\tilde{\lambda}^2}b^2(\log n)^2 + \left(\frac{2n^b}{\lambda^2} + \frac{n}{n^{5b}\tilde{\lambda}^2}\right)\frac{\pi^2}{6}}{2\left((2-b)\frac{n^b}{\lambda}\log n + \frac{n}{n^{3b}\tilde{\lambda}}b\log n\right)}\\&+\frac{b(2-b)\frac{n}{n^{2b}\lambda\tilde{\lambda}}(\log n)^2 + \frac{n^{2b}}{\lambda^2}(1-b)(\log n)^2}{(2-b)\frac{n^b}{\lambda}\log n + \frac{n}{n^{3b}\tilde{\lambda}}b\log n}. \quad (20)\end{aligned}$$

Proof: The expression follows upon substituting $M = n^b$ in (15) and noting that for large n, we have $H_n \approx \log n$. Further, G_n is monotonically increasing and converges to $\frac{\pi^2}{6}$. Since we have $M = n^b$, as n grows large M does too, resulting in $H_M \approx b\log n$ and G_M converging to $\frac{\pi^2}{6}$. We note that the notation $\approx$ in this theorem is mainly due to $\approx$ in H_n. ■

Theorem 3: For large n, and for $\frac{1}{4} \leq b \leq 1$, the average age of an S-D pair Δ given in (20) reduces to,

$$\Delta \approx cn^b\log n, \quad (21)$$

with a constant c. That is, age is $O(n^b\log n)$, for $\frac{1}{4} \leq b \leq 1$.

Proof: By analyzing the result of Theorem 2 we note that the first and third terms are $O(n^b\log n)$, and the second term is $O(n^{1-3b}\log n)$, and fourth term is a constant independent of n. The fifth term can be written as

$$\begin{aligned}&\frac{n^{2b}(\log n)^2\left(\frac{1+(1-b)^2}{\lambda^2} + \frac{b^2}{n^{2(4b-1)}\tilde{\lambda}^2})\right)}{n^b\log n\left(\frac{2(2-b)}{\lambda} + \frac{2b}{\tilde{\lambda}n^{4b-1}}\right)}\\&+\frac{n^{2b}(\log n)^2\frac{\pi^2}{6}\left(\frac{2}{n^b(\log n)^2\lambda^2} + \frac{1}{n^{7b-1}(\log n)^2\tilde{\lambda}^2}\right)}{n^b\log n\left(\frac{2(2-b)}{\lambda} + \frac{2b}{\tilde{\lambda}n^{4b-1}}\right)} \quad (22)\end{aligned}$$

which is $O(n^b\log n)$ when $b \geq 1-3b$. Continuing similarly for the remaining term shows that it is also $O(n^b\log n)$ which gives the overall scaling result for $\frac{1}{4} \leq b \leq 1$. ■

Thus, the proposed transmission scheme, which involves intra-cell cooperation and inter-cell MIMO-like transmissions of mega update packets, allows the successful communication of n S-D pairs, and achieves an average age scaling of $O(n^{\frac{1}{4}}\log n)$ per-user.

In the next section, we propose introducing hierarchy to the proposed three-phase transmission scheme to improve the average age scaling.

V. THREE-PHASE TRANSMISSION SCHEME WITH HIERARCHY

A. Motivation and Outline of the Scheme

The three-phase transmission scheme proposed in Section IV allows successful communication of n S-D pairs. Following the analysis to obtain the average age expression by substituting the first and second order moments of the phase durations given in (16)-(19) into the average age expression given in (8), we observe that the resulting per-user average age scaling, when n is large, with $M = n^b$ where $0 < b \leq 1$ and exponential link delays, is characterized by the expected scaling of the phases. As derived in (16)-(18) expected durations of the phases are $O(n^b\log n)$, $O(n^{1-3b}\log n)$ and $O(n^b\log n)$ which in turn result in an average age scaling of $O(n^{\frac{1}{4}}\log n)$ upon selecting $b = 1-3b$. Thus, to obtain a better average age scaling we need to improve the expected length of each phase. This motivates the hierarchical cooperation in the proposed scheme.

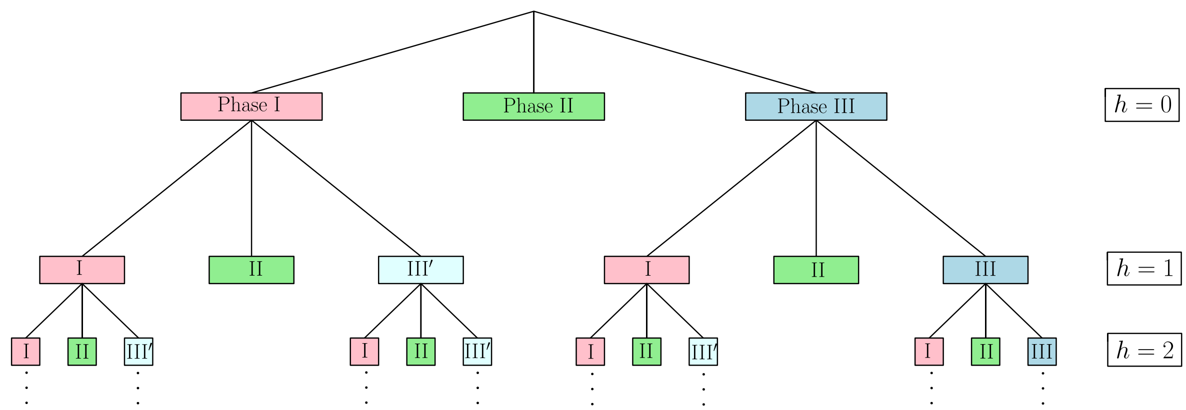

Fig. 4. Proposed three-phase hierarchical transmission scheme.

TABLE I

COMPARISON OF THE EXPECTED DURATIONS OF THE PHASES WITH $h = 0$ AS IN SECTION IV AND $h = 1$ HIERARCHY LEVEL WITH $0 < a < b \leq 1$

	$h = 0$	$h = 1$
Phase I	$O(n^b \log n)$	$O(n^a \log n)$ + $O(n^{b-3a} \log n)$ + $O(n^{b-a} \log n)$
Phase II	$O(n^{1-3b} \log n)$	$O(n^{1-3b} \log n)$
Phase III	$O(n^b \log n)$	$O(n^a \log n)$ + $O(n^{b-3a} \log n)$ + $O(n^a \log n)$

In the Phase I of Section IV, the communication takes place in between $M = n^b$ nodes rather than n nodes and a simple TDMA operation is performed among these nodes which results in an average scaling of $O(n^b \log n)$. Instead, we introduce the first level of hierarchy by dividing each of these cells into n^{b-a} further subcells with n^a users each where $0 < a < b$. Then, we apply the same three-phase scheme with one difference to this cell to accommodate Phase I transmissions of Section IV. In particular, to create the mega packet of the cell, first local communication among the nodes is performed within subcells and MIMO-like transmissions are carried out in between subcells within a cell. Then, instead of relaying the received packet to a single node as in Phase III of Section IV, received packets are relayed to every other node in the subcell to create the mega update packet. With this operation, Phase I of Section IV is completed in three phases, Phase I, Phase II, Phase III′, each of which are scaled down versions of the overall scheme with the corresponding difference in the third phase which is denoted as Phase III′ to highlight this difference. The expected length of the first phase with $h = 1$ level of hierarchy is then $O(n^a \log n) + O(n^{b-3a} \log n) + O(n^{b-a} \log n)$ (see Section V-B for a detailed derivation) all of which are smaller than $O(n^b \log n)$ achieved in Section IV.

Similarly, Phase III of Section IV can also be completed in three phases under $h = 1$ level of hierarchy. However, this time in the third step we need Phase III rather than Phase III′ since we need to relay the received update packet within subcell to its intended recipient node to conclude the delivery.

Fig. 4 shows the proposed hierarchy structure in which Phases I and III of level h can be performed by applying the three-phase scheme on a smaller scale at level $h + 1$ accordingly. The advantage of the hierarchical transmission is summarized in Table I.

B. Detailed Description of the Scheme for $h = 1$

In this section, we describe the proposed hierarchical transmission scheme with $h = 1$ level of hierarchy in detail. Later, we generalize the average age scaling result for $h > 1$ levels of hierarchy using the fact that the system is scale-invariant. We note that the scheme in Section IV does not utilize hierarchical cooperation, i.e., $h = 0$. As in Section IV, we start with a square network that is divided into $\frac{n}{M}$ cells of equal area with M nodes in each cell with high probability that tends to 1 as n increases. Selecting $M = n^b$ where $0 < b \leq 1$ results in n^{1-b} equal area cells with n^b users each cell. Introducing the first level of hierarchy, we further divide each cell into n^{b-a} equal area subcells to get a total of n^{1-a} subcells with n^a nodes each where $0 < a < b$.

Remember that when there is no hierarchical cooperation we denote the transmission delays within cells as X_i and in between cells as $\tilde{X}_i$. In this section, in order to accommodate hierarchical structure for $h = 1$ level, we change our notation so that transmission delays between the nodes from different cells are now denoted by $X_i^{(0)}$, between the nodes from different subcells within the same cell are denoted by $X_i^{(1)}$, and between the nodes belonging to the same subcell are denoted by $X_i^{(2)}$. Note that $X_i^{(j)}$ are independent; and $X_i^{(j)}$ are i.i.d. exponential with parameter λ_j for $j = 0, 1, 2$. Note that we have $\lambda_2 \geq \lambda_1 \geq \lambda_0$ so that link delays are proportional to the distances between nodes on average. Also note that in what follows we use Y_I, Y_{II} and Y_{III} to denote the phase durations under the three-phase scheme with hierarchy.

1) Phase I. Creating Mega Update Packets: In this phase, each cell generates its mega update packet which includes all $M = n^b$ messages to be sent from that cell. Unlike the scheme in Section IV, we create mega update packets in three successive phases by applying the three-phase transmission scheme to each cell.

First, each node in a subcell distributes its update packet to remaining $n^a - 1$ nodes in its subcell which takes $U^I = X^{(2)}_{n^a-1:n^a-1}$ units of time. Considering n^a successive transmissions for each node of the subcell, this step is completed in a subcell in $V^I = \sum_{i=1}^{n^a} U_i^I$ units of time. This operation is analogous to the Phase I in Section IV but performed among n^a nodes in a subcell rather than among n^b nodes within a cell. Upon completion of this step in a subcell, each node of that subcell has n^a different update packets one from each node. Each node combines all these update packets to form a *preliminary* mega update packet which includes all n^a messages to be sent out from this subcell. This operation is performed in parallel among all subcells in the network (see Section VI for a detailed description of this operation) and ends when the slowest simultaneously operating subcell finishes creating its *preliminary* mega update packet. Hence, it takes $Y_I^I = V^I_{n^{1-a}:n^{1-a}}$ units of time, where Y_I^I denotes the duration of the first phase at $h = 1$.

When all *preliminary* mega update packets are formed, all n^{b-a} subcells of a cell perform MIMO-like transmissions among each other to distribute their *preliminary* mega update packets to remaining subcells within the cell. Since this requires cell-level transmissions in between subcells, this step is performed in parallel among cells and thus, subcells take turns. As in the Phase II of Section IV, all n^a nodes of a subcell start transmitting the *preliminary* mega update packet to remaining $n^{b-a} - 1$ subcells. Since every node sends the same *preliminary* mega update packet this does not create interference. This transmission continues until the earliest node in each remaining subcell receives the *preliminary* mega update packet. In other words, among the n^{2a} active links between the source and destination subcells, only the one with the smallest delay is critical. Thus, for a single subcell it takes $U^{II} = (X^{(1)}_{1:n^{2a}})_{n^{b-a}-1:n^{b-a}-1}$ units of time. Since subcells take turns, in a cell this step is completed in $V^{II} = \sum_{i=1}^{n^{b-a}} U_i^{II}$ units of time. Finally, on the network-level these MIMO-like transmissions continue until the slowest of the simultaneously operating cells finishes which corresponds to $Y_I^{II} = V^{II}_{n^{1-b}:n^{1-b}}$.

By the end of the MIMO-like transmissions among subcells, each subcell receives *preliminary* mega update packets of remaining $n^{b-a} - 1$ subcells that lie in its cell. In this step, these packets are distributed within the subcell in parallel among the subcells of the network. This is identical to the operation of Phase III of Section IV on subcell-level except that each *preliminary* mega update packet received is transmitted to all nodes of that subcell to successfully form the mega update packet of the corresponding cell. To highlight this difference we denote this step as Phase III′ in Fig. 4 at $h = 1$ level. Distributing one *preliminary* mega update packet takes $U^{III'} = X^{(2)}_{n^a-1:n^a-1}$ units of time. By repeating this for all *preliminary* mega update packets received this step is completed in a subcell in $V^{III'} = \sum_{i=1}^{n^{b-a}-1} U_i^{III'}$ units of time. We wait for the slowest subcell and thus on the network-level this step is completed in $Y_I^{III'} = V^{III'}_{n^{1-a}:n^{1-a}}$ units of time.

With this, each node in a subcell receives remaining $n^{b-a}-1$ *preliminary* mega update packets of n^a message each. Combining these with their own *preliminary* mega update packet, every node in a subcell forms the mega update packet which includes all n^b messages to be sent out from that cell. Thus, the first phase lasts for $Y_I = Y_I^I + Y_I^{II} + Y_I^{III'}$ units of time.

2) Phase II. MIMO-Like Transmissions: Identical to Phase II of Section IV, in this phase each cell successively performs MIMO-like transmissions using the mega update packets created in Phase I. This phase requires network-level transmissions between cells. Thus, only one cell operates at a time. As in Section IV, a source node s from cell j needs $\tilde{U} = X^{(0)}_{1:n^{2b}}$ units of time to send its update to the destination cell where the destination node d lies in. Transmissions of cell j continue until all n^b destination cells receive the mega update packet. Hence, for each cell, this phase lasts for $\tilde{V} = \tilde{U}_{n^b:n^b}$. This operation is repeated for each cell and hence the session time of this phase $Y_{II} = \sum_{i=1}^{n^{1-b}} \tilde{V}_i$. At the end of this phase, each cell delivers its mega update packet to one node in each of the corresponding destination cells.

3) Phase III. In-Cell Relaying to the Destination Nodes: By the end of Phase II, each cell receives a total of n^b mega update packets, one for each node. In Section IV, relevant packets which have a destination node in that cell are extracted from these mega update packets and relayed to their respective designated recipient nodes by a simple TDMA operation which scales as $O(n^b \log n)$. However, as in Phase I we can introduce hierarchy to this phase and apply the three-phase scheme again. Thus, extracted relevant packets are first distributed within subcells of the nodes which received them in Phase II. Then, these packets are delivered to their corresponding destination subcells in which the destination nodes are located through MIMO-like transmissions and finally, they are relayed to the corresponding recipient nodes within subcells.

Noting that each subcell receives on average n^a mega update packets, with one relevant packet each, distribution of these n^a packets within subcell takes $\hat{V}^I = \sum_{i=1}^{n^a} \hat{U}_i^I$ with $\hat{U}^I = X^{(2)}_{n^a-1:n^a-1}$ and on the network-level is completed in $Y^I_{III} = \hat{V}^I_{n^{1-a}:n^{1-a}}$ units of time. With this operation, the *secondary* mega update packet of that subcell is formed which includes all n^a update packets with destinations in that cell. Then, these *secondary* mega update packets are transmitted to the respective destination subcells in parallel among cells (subcells take turns) through MIMO-like transmissions until all n^a destination subcells receive them. In a cell, this is completed in $\hat{V}^{II} = \sum_{i=1}^{n^{b-a}} \hat{U}_i^{II}$ units of time where $\hat{U}^{II} = (X^{(1)}_{1:n^{2a}})_{n^a:n^a}$ and therefore, on the network-level is completed in $Y^{II}_{III} = \hat{V}^{II}_{n^{1-b}:n^{1-b}}$ when all cells finish. Thus, each subcell receives a total of n^a *secondary* mega update packets each of which includes one update destined to a node in that subcell. Finally, these packets are relayed to their actual recipient nodes within subcell. For a subcell it takes $\hat{V}^{III} = \sum_{i=1}^{n^a} \hat{U}_i^{III}$ units of time where $\hat{U}^{III} = X^{(2)}$ and hence on the network-level it is completed in $Y^{III}_{III} = \hat{V}^{III}_{n^{1-a}:n^{1-a}}$. Note that since in the last step we relay the packets to their destination node rather than all nodes in the subcell, this step is the subcell-level equivalent of Phase III of Section IV. As a result, the third phase lasts for $Y_{III} = Y_{III}^{I} + Y_{III}^{II} + Y_{III}^{III}$ and finishes when every S-D pair of the network is served.

Total session time of the proposed scheme is, therefore, $Y = Y_I + Y_{II} + Y_{III}$. Before we perform the explicit age calculation, we again make some observations to simplify our analysis.

Lemma 3: Y_I satisfies the following inequality,

$$Y_I \leq \bar{V}^I + \bar{V}^{II} + \bar{V}^{III'}, \tag{23}$$

where

$$\bar{V}^I = \sum_{i=1}^{n^a} \bar{U}_i^I, \quad \bar{U}^I = X_{n:n}^{(2)}, \tag{24}$$

$$\bar{V}^{II} = \sum_{i=1}^{n^{b-a}} \bar{U}_i^{II}, \quad \bar{U}^{II} = (X_{1:n^{2a}}^{(1)})_{n^{1-a}:n^{1-a}}, \tag{25}$$

$$\bar{V}^{III'} = \sum_{i=1}^{n^{b-a}} \bar{U}_i^{III'}, \quad \bar{U}^{III'} = X_{n:n}^{(2)}. \tag{26}$$

The proof of this lemma follows similarly from that of Lemma 1. We show that $Y_I^I \leq \bar{V}^I$, $Y_I^{II} \leq \bar{V}^{II}$ and $Y_I^{III'} \leq \bar{V}^{III'}$ which yields (23).

We worsen our scheme in terms of session time and hereafter take the upper bound in Lemma 3 as our Phase I duration for tractability and ease of calculation. Thus, from now on $Y_I = \bar{V}^I + \bar{V}^{II} + \bar{V}^{III'}$. Next, we have the following upper bound for the duration of Phase III.

Lemma 4: Y_{III} satisfies the following inequality,

$$Y_{III} \leq \bar{\bar{V}}^I + \bar{\bar{V}}^{II} + \bar{\bar{V}}^{III}, \tag{27}$$

where

$$\bar{\bar{V}}^I = \sum_{i=1}^{n^a} \bar{\bar{U}}_i^I, \quad \bar{\bar{U}}^I = X_{n:n}^{(2)}, \tag{28}$$

$$\bar{\bar{V}}^{II} = \sum_{i=1}^{n^{b-a}} \bar{\bar{U}}_i^{II}, \quad \bar{\bar{U}}^{II} = (X_{1:n^{2a}}^{(1)})_{n^{1-b+a}:n^{1-b+a}}, \tag{29}$$

$$\bar{\bar{V}}^{III} = \sum_{i=1}^{n^a} \bar{\bar{U}}_i^{III}, \quad \bar{\bar{U}}^{III} = X_{n^{1-a}:n^{1-a}}^{(2)}. \tag{30}$$

We omit the proof of Lemma 4 since it follows similar to the proof of Lemma 1. We worsen Phase III as well in terms of duration and take $Y_{III} = \bar{\bar{V}}^I + \bar{\bar{V}}^{II} + \bar{\bar{V}}^{III}$ from now on because of similar tractability issues.

As a result of Lemmas 3 and 4, total session time becomes

$$Y = \bar{V}^I + \bar{V}^{II} + \bar{V}^{III'} + Y_{II} + \bar{\bar{V}}^I + \bar{\bar{V}}^{II} + \bar{\bar{V}}^{III} \tag{31}$$

Taking expectations of order statistics of exponential random variables as in (2)-(4) and using the fact that for large n, we have $H_n \approx \log n$ and G_n is monotonically increasing and converges to $\frac{\pi^2}{6}$, first two moments of the subphase and phase durations approximately become

$$\mathbb{E}\left[\sum_{i\in\mathcal{I}'} \bar{V}^{(i)}\right] = \left(\frac{n^a + n^{b-a}}{\lambda_2} + \frac{(1-a)n^{b-3a}}{\lambda_1}\right)\log n, \tag{32}$$

$$\mathbb{E}\left[\sum_{i\in\mathcal{I}} \bar{\bar{V}}^{(i)}\right] = \left(\frac{(2-a)n^a}{\lambda_2} + \frac{(1-b+a)n^{b-3a}}{\lambda_1}\right)\log n, \tag{33}$$

$$\mathbb{E}[Y_{II}] = \frac{bn^{1-3b}}{\lambda_0}\log n, \tag{34}$$

$$\mathbb{E}\left[Y_{II}^2\right] = \frac{n^{1-5b}}{\lambda_0^2}\frac{\pi^2}{6} + \frac{b^2 n^{2(1-3b)}}{\lambda_0^2}\log^2 n, \tag{35}$$

$$\mathbb{E}\left[\left(\bar{V}^I\right)^2\right] = \frac{n^a}{\lambda_2^2}\frac{\pi^2}{6} + \frac{n^{2a}}{\lambda_2^2}\log^2 n, \tag{36}$$

$$\mathbb{E}\left[\left(\bar{V}^{II}\right)^2\right] = \frac{n^{b-5a}}{\lambda_1^2}\frac{\pi^2}{6} + \frac{(1-a)^2 n^{2(b-3a)}}{\lambda_1^2}\log^2 n, \tag{37}$$

$$\mathbb{E}\left[\left(\bar{V}^{III'}\right)^2\right] = \frac{n^{b-a}}{\lambda_2^2}\frac{\pi^2}{6} + \frac{n^{2(b-a)}}{\lambda_2^2}\log^2 n, \tag{38}$$

$$\mathbb{E}\left[\left(\bar{\bar{V}}^I\right)^2\right] = \frac{n^a}{\lambda_2^2}\frac{\pi^2}{6} + \frac{n^{2a}}{\lambda_2^2}\log^2 n, \tag{39}$$

$$\mathbb{E}\left[\left(\bar{\bar{V}}^{II}\right)^2\right] = \frac{n^{b-5a}}{\lambda_1^2}\frac{\pi^2}{6} + \frac{(1-b+a)^2 n^{2(b-3a)}}{\lambda_1^2}\log^2 n, \tag{40}$$

$$\mathbb{E}\left[\left(\bar{\bar{V}}^{III}\right)^2\right] = \frac{n^a}{\lambda_2^2}\frac{\pi^2}{6} + \frac{(1-a)^2 n^{2a}}{\lambda_2^2}\log^2 n, \tag{41}$$

where in (32), $i \in \mathcal{I}' = \{I, II, III'\}$ and in (33), $i \in \mathcal{I} = \{I, II, III\}$.

Now, we are ready to derive an average age expression using (8). For ease of exposition, we assume that every node updates its age at the end of each session when the hierarchy is implemented and take $D_{j+1} = Y_{j+1}$. Then, (8) becomes

$$\Delta = E[Y] + \frac{E[Y^2]}{2E[Y]}. \tag{42}$$

Note that this assumption can only result in a higher average age as all nodes but one receive their update packets before the session ends, i.e., $P(D \leq Y) = 1$ for all updates and nodes.

Theorem 4: Under the constructed transmission scheme with $h = 1$ level of hierarchy, for large n, the average age of an S-D pair is given by,

$$\Delta = \mathbb{E}\left[\sum_{i\in\mathcal{I}'} \bar{V}^{(i)}\right] + \mathbb{E}[Y_{II}] + \mathbb{E}\left[\sum_{i\in\mathcal{I}} \bar{\bar{V}}^{(i)}\right] + \frac{\mathbb{E}\left[\left(\sum_{i\in\mathcal{I}'} \bar{V}^{(i)} + Y_{II} + \sum_{i\in\mathcal{I}} \bar{\bar{V}}^{(i)}\right)^2\right]}{2\left(\mathbb{E}\left[\sum_{i\in\mathcal{I}'} \bar{V}^{(i)}\right] + \mathbb{E}[Y_{II}] + \mathbb{E}\left[\sum_{i\in\mathcal{I}} \bar{\bar{V}}^{(i)}\right]\right)}. \tag{43}$$

The proof of Theorem 4 follows upon substituting (31) back in (42). Moments follow from (32)-(41).

Theorem 5: For large n, with $a = \frac{b}{2}$ and $\frac{1}{7} \leq a \leq \frac{1}{2}$, the average age of an S-D pair when $h = 1$ hierarchy level is implemented, Δ, given in (43) reduces to,

$$\Delta \approx \tilde{c} n^a \log n, \tag{44}$$

with a constant $\tilde{c}$. That is, age is $O(n^a \log n)$, for $\frac{1}{7} \leq a \leq \frac{1}{2}$.

Proof: Using (32)-(41) in (43), we observe that in the average age expression we have terms with $O(n^a \log n)$, $O(n^{b-a} \log n)$, $O(n^{b-3a} \log n)$, and $O(n^{1-3b} \log n)$. Among first three types, noting that $b - 3a < b - a$, dominating terms are $O(n^a)$ and $O(n^{b-a})$. Thus, by choosing $a = b - a$ we can minimize the resulting scaling. With this selection, the first and third terms in (43) are $O(n^a \log n)$ whereas the second one is $O(n^{1-6a} \log n)$. Looking at the fourth term we observe that it has the following form when $b = 2a$,

$$\frac{c_1 n^{2a} \log^2 n + c_2 n^{2(1-6a)} \log^2 n + c_3 n^{1-5a} \log^2 n}{c_4 n^a \log n + c_5 n^{1-6a} \log n}, \quad (45)$$

where $c_1, \dots, c_5$ are constants. We observe that for $\frac{1}{7} \leq a \leq \frac{1}{2}$, first three terms and the fourth term given in (45) are $O(n^a \log n)$ which yields the result. ∎

Thus, the proposed hierarchical scheme with $h = 1$ hierarchy levels achieves an average age scaling of $O(n^{\frac{1}{7}} \log n)$ per-user when $a = \frac{1}{7}$ and $b = \frac{2}{7}$. This implies that if the cells have M nodes each, each subcell has $\sqrt{M}$ nodes when $h = 1$. Note that in Section IV it is shown that $\frac{1}{4} \leq b \leq 1$. Here, resulting b not only satisfies this but also gives a better scaling in the end because of the hierarchy we utilized. In Theorem 6 below, we generalize this scaling result to h levels of hierarchy.

Theorem 6: For large n, when the proposed scheme is implemented with h hierarchy levels, the average age scaling of $O\left(n^{\alpha(h)} \log n\right)$ per-user is achievable where $\alpha(h) = \frac{1}{3 \cdot 2^h + 1}$.

Proof: We observe that when $h = 1$ hierarchy level is utilized, the scaling result comes from $a = 1 - 6a$. Since $b = 2a$, another way to express this is $\frac{b}{2^h} = 1 - 3b$. As h increases with $b = 2a$ structure in each hierarchy level, we see that subcells at level h have $n^{\frac{b}{2^h}}$ nodes. Thus, when h levels of hierarchy is utilized, subcell transmissions take place among $n^{\frac{b}{2^h}}$ nodes and inter-subcell transmissions have $n^{\frac{b}{2^h}}$ turns. However, the second phase is still $O(n^{1-3b})$ as each cell at the top of the hierarchy has n^b nodes. Thus, $\frac{b}{2^h} = 1 - 3b$ yields $\alpha(h) = \frac{1}{3 \cdot 2^h + 1}$. ∎

Thus, when hierarchy is utilized, the proposed transmission scheme, which involves local cooperation and MIMO-like inter-cell transmissions, allows the successful communication of n S-D pairs, and achieves an average age scaling of $O\left(n^{\alpha(h)} \log n\right)$ per-user where $h = 0, 1, \dots$ is the number of hierarchy levels. Note that in the asymptotic case when h tends to ∞, this scheme gives an average age scaling of $O(\log n)$ per-user. This is the case because as h increases, number of turns in each phase, $n^{\frac{b}{2^h}}$, decreases such that eventually $\log n$ term which comes from the fact that packets are distributed locally to all other nodes in the same subcell in Phases I and III$'$ dominates. We also observe that when the hierarchy is not utilized, i.e. $h = 0$, Theorem 6 yields the result in Theorem 3 in Section IV.

VI. Note on Phases I and III

We use the protocol model introduced in [48] to model the interference such that two nodes can be active if they are sufficiently spatially separated from each other. In other words, we allow simultaneous transmissions provided there is no destructive interference caused by other active nodes. Suppose that node i transmits its update to node j. Then, node j can successfully receive this update if the following is satisfied for any other node k that is simultaneously transmitting,

$$d(j, k) \geq (1 + \gamma) d(j, i), \quad (46)$$

where function $d(x, y)$ denotes the distance between nodes x and y and γ is a positive constant determining the guard zone.

The proposed three-phase scheme with h levels of hierarchy utilizes parallelized transmissions in Phases I and III where $h \geq 0$. When the hierarchy is not utilized, i.e., $h = 0$, parallel intra-cell transmissions take place during these phases. In order to implement these parallel communications in Phases I and III, we follow a 9-TDMA scheme as in [53]. Specifically, $O(\frac{n}{9M})$ of the total $\frac{n}{M}$ cells work simultaneously so that Phases I and III are completed in 9 successive subphases. Using the protocol model, cells that are at least $(1 + \gamma) r \sqrt{2}$ away from a cell can operate simultaneously during these phases, where $r = \sqrt{SM/n}$ is the length of each square cell and S is the network area. Noting that there are at least two inactive cells in between two active cells under a 9-TDMA operation and the maximum in-cell transmission distance is $r\sqrt{2}$, this scheme satisfies (46) if the guard zone parameter $\gamma \leq \sqrt{2} - 1$. We note that as the number of cells in the network increases, the number of simultaneously active nodes in Phases I and III also increases. Since the distance between the active cells, $2r$, and the in-cell transmission distance, $r\sqrt{2}$, both decrease proportionally when the number of cell increases the condition in (46) is still satisfied under the 9-TDMA operation given that we have $\gamma \leq \sqrt{2} - 1$.

On the other hand, when $h = 1$ level of hierarchy is utilized, the proposed scheme includes within subcell transmissions that are parallelized across subcells and within cell transmissions that are parallelized among cells (subcells take turns) in Phases I and III (or III$'$). Similar 9-TDMA scheme again is used to accommodate these simultaneous transmissions. When $\gamma \leq \sqrt{2} - 1$, parallel 9-TDMA operation among subcells is still allowed since from cell-level to subcell-level both distance terms in (46) decrease proportionally. Extending this, we see that for h level of hierarchy, by selecting an appropriate guard zone parameter γ, parallelized Phase I and III (or III$'$) operation under 9-TDMA scheme is allowed. Noting that 9 here is a constant and valid for any n, it does not change the scaling results.

VII. Numerical Results

In this section, we provide simple numerical results to validate our results for the $h = 0$ case, i.e., hierarchical cooperation is not utilized.[6] Simulations are performed using MATLAB over 1000 sessions where each session is comprised of three phases. Plotted results are averaged over 10 independent simulations. In the simulations, we set $\lambda = 5$ and $\tilde{\lambda} = 2$. We recall that to make the proposed three-phase scheme analytically tractable, we worsen Phases I and III through Lemmas 1 and 2. In this section, we provide simulations for the actual proposed three-phase scheme, referred to as TPS and shown in blue solid curves throughout, and its upper-bounded version, referred to as TPS-ub and shown in purple dash-dotted curves throughout, along with our theoretical results to make comparisons and verify our results.

[6] Simulation complexity increases substantially when hierarchical cooperation is utilized.

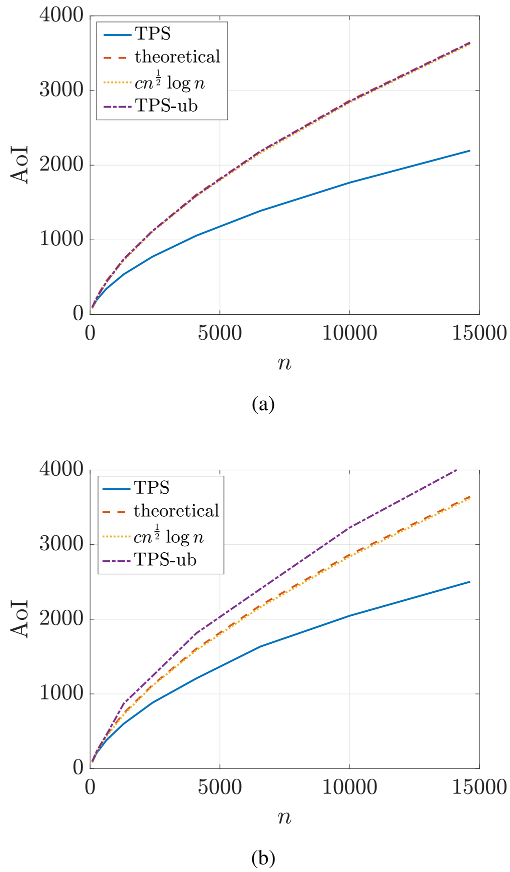

Fig. 5. Average age scaling under the proposed three-phase hierarchical transmission scheme for $h = 0$, $b = \frac{1}{2}$, $\lambda = 5$ and $\tilde{\lambda} = 2$ for varying n when (a) nodes are on a grid, (b) nodes are randomly uniformly and independently distributed.

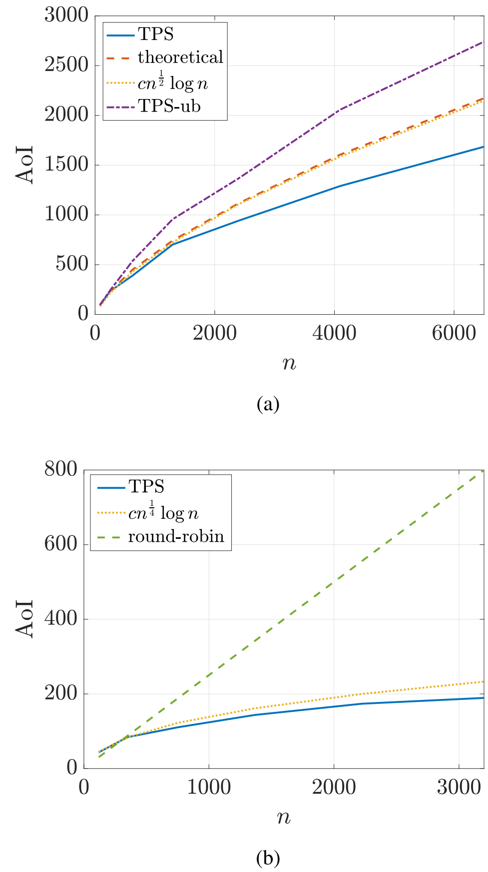

Fig. 6. (a) Average age scaling under the proposed three-phase hierarchical transmission scheme for $h = 0$, $b = \frac{1}{2}$ when nodes have non i.i.d. transmission rates for varying n. (b) Comparison of round robin scheme and the proposed three-phase hierarchical transmission scheme for $h = 0$, $b = \frac{1}{4}$, $\lambda = 5$ and $\tilde{\lambda} = 2$ for varying n.

We first consider the case in which the nodes are placed on a grid in the network and set $b = \frac{1}{2}$. In other words, nodes are equally spaced and each of the $\frac{n}{M}$ cells has exactly M nodes. We note that since we do not consider the physical distance in between nodes in our analysis, our results still hold when the nodes are on a grid. In Fig. 5(a), in line with Theorem 3, we see that TPS-ub achieves an average age scaling of $O(n^{\frac{1}{2}} \log n)$ per-user since we set $b = \frac{1}{2}$. Here, the red dashed curve shows the theoretical result obtained from (16)-(18) whereas the yellow dotted line is obtained from (21) with corresponding c. We observe that these two curves coincide, even for smaller values of n. Thus, the result in (21) is in line with our average age analysis. Further, we observe that the actual proposed policy without the upper bounds on Phases I and III, TPS, lies below the $O(n^{\frac{1}{2}} \log n)$ scaling. Thus, the proposed three-phase scheme, without the upper bounds on Phases I and III, has a better average age scaling than the upper bounded scheme, TPS-ub, which is shown to achieve $O(n^{\frac{1}{2}} \log n)$ scaling for $b = \frac{1}{2}$ in Theorem 3.

Next, we consider the case in which the nodes are uniformly and independently placed in the network in Fig. 5(b). In this case, each cell has $O(M)$ nodes rather than exactly M nodes which leads to a gap in between the TPS-ub curve and the theoretical results, which assume exactly M nodes in each cell, even though we observe that TPS-ub curve continues to have $O(n^{\frac{1}{2}} \log n)$ scaling trend. Thus, for scaling results to hold, it is enough to have $O(M)$ nodes in each cell. We also observe that age under the actual proposed scheme, TPS, slightly increases compared to Fig. 5(a) but still has a better scaling than $O(n^{\frac{1}{2}} \log n)$.

Throughout the analysis, we have i.i.d. transmission times for intra-cell and inter-cell transmissions of each node. Next, we analyze the performance of the proposed scheme when nodes have different transmission rates. Thus, in this case, transmission times of nodes are independently yet exponentially distributed with different mean values. In the simulation we consider that λ is uniformly distributed in $[3, 7]$ and $\tilde{\lambda}$ is uniformly distributed in $[0.5, 3.5]$ for each node such that average intra-cell and inter-cell transmission rates are 5 and 2, respectively as in Fig. 5(b). We observe in Fig. 6(a) that under the non-i.i.d. transmission times, average ages for both TPS and TPS-ub increase even though the TPS curve still has a lower average age scaling than $O(n^{\frac{1}{2}} \log n)$. On the other hand, in this case, the gap between the $cn^{\frac{1}{2}} \log n$ curve and TPS-ub curve increases and these two curves are no longer parallel. That is, TPS-ub has a higher average age scaling than $O(n^{\frac{1}{2}} \log n)$ unlike the i.i.d. transmission rates setting.

Lastly, we compare the performance of the three-phase scheme with $b = \frac{1}{4}$ and the baseline round robin policy in which nodes take turns to transmit their update packets which has per-user scaling of $O(n)$. In Fig. 6(b), we observe the proposed three-phase transmission scheme outperforms the round-robin policy when the total number of nodes exceeds $n = 300$.

VIII. Discussion

We note that the focus of this paper is the scaling of age of information in large wireless networks. Thus, presented scaling results hold with high probability when the number of nodes in the network grows beyond a certain threshold. The random network model used in this work is rather idealized. Although it is of theoretical interest on its own right, a more realistic model may adopt a physical interference model that is based on signal-to-interference ratio requirements and possibly mobile nodes rather than static nodes. Moreover, we make use of the mega update packets in the proposed transmission scheme to serve multiple S-D pairs at once without considering the growing mega update packet size as the network population increases. Likewise, link delays are modeled as i.i.d. exponentials with constant parameters that are not affected by the transmission distance or packet size. It would be an interesting direction to analyze the effects of packet size and the distance between S-D pairs on the average age scaling. Further, the schemes discussed in this paper are not private since a packet intended for a certain destination node is observed by other nodes in the network. To ensure privacy, updates can be encrypted in a way that the routing of a packet to the correct destination node is still maintained but only the intended destination node can fully decrypt the message.

In this sense, here, we take a careful look at the proposed three-phase transmission scheme, particularly the MIMO-like inter-cell transmissions of mega update packets in Phase II, and discuss the performance of the proposed three-phase transmission scheme under possibly more realistic and practically applicable network models.

First, we analyze the performance of the proposed three-phase scheme without the utilization of mega update packets in the next corollary.

Corollary 1: When mega update packets are not utilized, the proposed three-phase transmission policy achieves an average age scaling of $O(n^{\frac{1}{3}}\log n)$ per-user.

Proof: In the proposed scheme, mega update packets are formed during Phase I and are transmitted to destination cells in Phase II. By transmitting a mega update packet, all M nodes of a particular cell send their messages to the corresponding destination cells at once. When the mega update packets are not utilized, however, each node of a cell still stores all other messages received in Phase I in its buffer but instead of combining these messages to create the mega update packet, these nodes can send the individual packets to corresponding M destination cells sequentially. In other words, rather than sending all M packets as a mega packet simultaneously, nodes in a cell collectively can send the packets one by one.[7] In this operation, a cell sends out all M of its update packets to the destination cells in M successive transmissions in $\sum_{i=1}^{M}(\tilde{X}_{1:M^2})_i$ units of time where $\tilde{X}$ denotes the transmission delay of a single packet and is exponentially distributed with rate $\tilde{\lambda}$.[8] Since we need a total of n transmissions for $\frac{n}{M}$ cells we have $Y_{II} = \sum_{i=1}^{n}(\tilde{X}_{1:M^2})_i$. Note that when $M = n^b$ where $0 < b \leq 1$ we have

$$\mathbb{E}[Y_{II}] = \frac{n}{M^2\tilde{\lambda}}, \quad \mathbb{E}[Y_{II}^2] = \frac{n^2}{M^4\tilde{\lambda}^2} + \frac{n}{M^4\tilde{\lambda}^2}. \tag{47}$$

Repeating a similar analysis as above with (47) instead of (17) for large n yields the result. Particularly, we obtain $b = 1-2b$ from which the proposed scheme achieves the $O(n^{\frac{1}{3}}\log n)$ scaling result. Note that even if the mega update packets are not utilized, nodes in a cell still need enough buffer space to store all M messages received in Phase I so that these messages can be sent out one by one in Phase II. ■

Second, in the next corollary, we consider the case when the speed up factor is M in the MIMO-like transmissions of Phase II as opposed to a speed up factor of M^2 which arises since we consider the fastest of the simultaneously active M^2 links in Phase II.

Corollary 2: When the speed up factor is M as in the ordinary MIMO with M transmit and M receive antennas, in the inter-cell transmissions of Phase II, the proposed three-phase transmission policy achieves an average age scaling of $O(n^{\frac{1}{3}}\log n)$ per-user.

Proof: In this case, we have $\tilde{U} = \tilde{X}_{1:M}$ such that the first two moments of the duration of the second phase become

$$\mathbb{E}[Y_{II}] = \frac{n}{M^2\tilde{\lambda}}H_M, \quad \mathbb{E}[Y_{II}^2] = \frac{n^2}{M^4\tilde{\lambda}^2}H_M^2 + \frac{n}{M^3\tilde{\lambda}^2}G_M, \tag{48}$$

with $M = n^b$ where $0 < b \leq 1$. Repeating a similar analysis as above with (48) instead of (17) for large n yields the result. ■

We note that a gain of M in the MIMO-like transmissions can be the result of highly correlated transmission times from a node in the source cell to M nodes in the destination cell in the physical layer. That is, all M links in between a node in the source cell and nodes in the destination cell have highly correlated transmission times such that we consider the fastest of the simultaneously active M links, one for each node in the source cell, in Phase II as opposed to the fastest of M^2 links. Further, we note that the proposed transmission scheme achieves a better age scaling than the delay scaling of [54] which also uses MIMO transmissions for inter-cell transmissions and achieves a delay scaling of $O(n^{\frac{1}{2}}\log n)$ per-user while sacrificing the throughput performance.

Third, we consider the case in which the mega update packet transmission rate depends explicitly on the number of packets in the mega updates, M. So far, the inter-cell transmissions of mega update packets are modeled by i.i.d. exponential random variables with rate $\tilde{\lambda}$. Since these mega update packets are

[7] We note that in this operation, each destination cell only receives the update packets that are destined to that particular cell as opposed to receiving a full mega update packet.

[8] We note that when mega update packets are not utilized, the transmission of all M update packets from a particular cell to corresponding M destination cells essentially has an Erlang distribution with rate $(M, M^2\tilde{\lambda})$ since we have sum of M i.i.d. random variables $\tilde{X}_{1:M^2}$ which is exponentially distributed with rate $M^2\tilde{\lambda}$.

comprised of M update packets, one for each node of a particular cell, here, we scale down the exponential random variable with the mega update packet size, M. That is, we let $\tilde{X}$ denote the transmission delay of a mega update packet such that the transmission delay is exponentially distributed with rate $\frac{\tilde{\lambda}}{M}$ instead of $\tilde{\lambda}$ and analyze the performance of the proposed three-phase transmission policy in the next corollary.

Corollary 3: When the mega update packet transmission delays, $\tilde{X}$ of Phase II are modeled by exponential random variables with rate $\frac{\tilde{\lambda}}{M}$, the proposed three-phase transmission policy achieves an average age scaling of $O(n^{\frac{1}{3}}\log n)$ per-user.

We note that, in this case, the first two moments of the duration of the second phase are again given by (48) which again yields the same result for large n.

Next, we consider the cases discussed in Corollaries 2 and 3 together. That is, we analyze the performance of the proposed three-phase scheme when the speedup factor is M, as in the ordinary MIMO scheme, with the inter-cell transmission of mega update packets modeled by scaled-down exponential random variables with the mega update packet size, M. In this case, the proposed scheme achieves a per-user scaling of $O(n^{\frac{1}{2}}\log n)$.

Lastly, we consider the case in which a single mega update packet transmission is modeled by an Erlang distribution with rate $(M, M^2\tilde{\lambda})$. That is, inter-cell transmission of a single packet has i.i.d. exponential delays with rate $\tilde{\lambda}$ such that MIMO-like inter-cell transmission of a single packet has an i.i.d. exponentially distributed delay with rate $M^2\tilde{\lambda}$. We note that this case is essentially equivalent to the case when mega update packets are not utilized, as discussed in Corollary 1, since when we have M successive transmissions for a mega packet transmission, then each packet is only sent to its destination cell. In other words, a destination cell receives only the packets that are destined to that particular cell as opposed to receiving the whole mega update packet. Thus, when the mega update packets have Erlang service times, the proposed three-phase policy achieves an average age scaling of $O(n^{\frac{1}{3}}\log n)$ per-user.

We note that in Corollaries 1 to 3, we consider the performance of the proposed scheme without invoking the hierarchical utilization and the corresponding hierarchical extensions of these settings differ from Section V and are not discussed within the scope of this paper.

IX. Conclusion

Given a large wireless network of fixed area consisting of n randomly located source-destination pairs that want to send time-sensitive status update packets to each other, we have studied the scalability of age of information. To accommodate the communication between the S-D pairs, we have proposed a three-phase transmission scheme which uses local cooperation between nodes and mega update packets to achieve an average age scaling of $O(n^{\frac{1}{4}}\log n)$. Our scheme divides the network into $\frac{n}{M}$ cells of M nodes each. The first and third phases include intra-cell transmissions and take place simultaneously across all cells. The second phase includes inter-cell transmissions and therefore during this phase cells operate one at a time.

We observe that the bottleneck in the resulting age scaling result is caused by $O(M)$ transmissions in Phases I and III. Furthermore, we note that each cell is a scaled-down version of the whole network. With these, we introduce hierarchy to the system and apply the three-phase scheme on a cell-level in these phases. In other words, Phases I and III of the hth level of the hierarchy are completed in three successive steps in the next level of hierarchy. We have shown that this scheme with hierarchical cooperation improves the scaling result and achieves an average age scaling of $O\left(n^{\alpha(h)}\log n\right)$ where $\alpha(h)=\frac{1}{3\cdot 2^h+1}$ and h is the number of hierarchy levels. In the asymptotic case when h tends to ∞ resulting average age per-user scales as $O(\log n)$.

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